

# The Mathematics of Power

## Weighted Voting

# Topic 02 // Lesson 01

O: to evaluate a weighted voting system

One person = one vote

One person =  $x$  vote(s)  **POWER?**

Different amount of votes mean different amounts of say.

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## weighted voting system

Any formal voting arrangement in which voters are not necessarily equal in terms of the number of votes they control.

## motions

“Yes – No votes”

→ those with only two choices

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## 3 Elements of every Weighted Voting Systems ▼

### 1. players

Voters in a weighted voting system

→ could be individuals, corporations, states, agencies, countries, etc

→ N: # of players

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## 3 Elements of every Weighted Voting Systems ▼

### 2. weights

The number of votes controlled by a player

→  $V$ : total # of votes

→  $w_1, w_2, w_3, \dots$

$$V = w_1 + w_2 + w_3 + \dots + w_N$$

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## 3 Elements of every Weighted Voting Systems ▼

### 3. quota

The minimum number of votes rerquired to pass a motion,  $q$

- Senate: simple majority for ordinary law
- Senate: 60 votes to stop filibuster
- Senate: 2/3s to override a presidential veto
- Sometimes: 3/4s or 100%

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► *Notation:*

$[q : w_1, w_2, w_3, \dots, w_N]$

↑                    ↑                    ↑                    ↑

quota                    weights

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## *Ex 2.1 Venture Capitalism*

Four partners ( $P_1$ ,  $P_2$ ,  $P_3$ , and  $P_4$ ) are starting a business.  $P_1$  buys 8 shares,  $P_2$  buys 7 shares,  $P_3$  buys 3 shares, and  $P_4$  buys 2 shares.

The quota is set at  $2/3s$ .

Use the Notation.



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## *Ex 2.2 Anarchy*

Four partners ( $P_1$ ,  $P_2$ ,  $P_3$ , and  $P_4$ ) are starting a business.  $P_1$  buys 8 shares,  $P_2$  buys 7 shares,  $P_3$  buys 3 shares, and  $P_4$  buys 2 shares.

The quota is set at 10.

What happens?

A: Math Anarchy

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## *Ex 2.3 Gridlock*

Four partners ( $P_1$ ,  $P_2$ ,  $P_3$ , and  $P_4$ ) are starting a business.  $P_1$  buys 8 shares,  $P_3$  buys 3 shares, and  $P_4$  buys 2 shares.

The quota is set at 21.

What happens?      A: Math Gridlock

$$V/2 < q \leq V$$

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*Ex 2.4 One Partner – One Vote*

[19: 8, 7, 3, 2]

The quota is set at 19.

What happens?

A: still needs 100% vote to pass. Its like a 1-1

A: impossibility = deception

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## *Ex 2.5 Dictators*

[11: 12, 5, 4]

The quota is set at 11.

What happens?

Dictator: a player whose weight is bigger than or equal to the quota.

Dummies: a player who has no power

\*a dictator is not necessary for there to be a dummy

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## *Ex 2.6 Unsuspecting Dummies*

[30: 10, 10, 10, 9]

The quota is set at  $3/4s$ .

What happens?

A:  $q = 30$ . this means last person with 9 has no power if other 3 make a decision. No real impact on votes

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*Ex 2.7 Veto Power*

[12: 9, 5, 4, 2]

The quota is set at 12.

What happens? A:  $P_1$  is a spoiler

A:  $P_1$  is not a dictator, but can stop a vote

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## *Ex 2.7 Veto Power*

Veto Power: a player who if they do not vote for a motion, then it cannot pass.

A player has veto power iff  
 $w < q$  and  $V - w < q$

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## Weighted Voting Practice

→ p.66 (1-10)