

Basic Elements of a Fair-Division Game

The underlying elements of every *fair-division game* are as follows:

The goods or “booty” “S”

This is the informal name we will give to the item or items being divided.

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In some situations the items being divided may be:

Intangible things: rights— water rights, drilling rights, broadcast licenses, etc

Things with a positive value: candy, cake, pizza, jewelry, art, land, and so on.

Things with a negative value: chores, liabilities, obligations, and so on.

Basic Elements of a Fair-Division Game

The players.

a set of parties with the right (or in some cases the duty) to share S

They are the players in the game. Most of the time the players in a fair-division game are individuals, but it is worth noting that some of the most significant applications of fair division occur when the players are institutions (ethnic groups, political parties, states, and even nations).

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The value systems.

The fundamental assumption we will make is that each player has an internalized value system that gives the player the ability to quantify the value of the booty or any of its parts.

Specifically, this means that each player can look at the set S or any subset of S and assign to it a value – either in absolute terms (“to me, that’s worth \$147.50”) or in relative terms (“to me, that piece is worth 30% of the total value of S ”).

Basic Assumptions

Like most games, fair-division games are predicated on certain assumptions about the players. For the purposes of our discussion, we will make the following four assumptions:

4 Basic Assumptions

1. Rationality

Each of the players is a thinking, rational entity seeking to maximize his or her share of the booty S . We will further assume that in the pursuit of this goal, a player's moves are based on reason alone

(we are taking emotion, psychology, mind games, and all other non-rational elements out of the picture.)

4 Basic Assumptions

2 Cooperation

The players are willing participants and accept the rules of the game as binding.

The rules are such that after a finite number of moves by the players, the game terminates with a division of S .

(There are no outsiders such as judges or referees involved in these games – just the players and the rules.)

Basic Assumptions

3 Privacy

Players have no useful information on the other players' value systems and what kinds of moves they are going to make in the game.

(This assumption does not always hold in real life, especially if the players are siblings or friends.)

Basic Assumptions

4 Symmetry

Players have equal rights in sharing the set S . A consequence of this assumption is that, at a minimum, each player is entitled to a proportional share of S – then there are two players, each is entitled to at least one-half of S , with three players each is entitled to at least one-third of S , and so on.

Fair Share

Objective: *assign shares to players in such a way that each player gets a fair share*

We can think of a *fair-division method* as the set of rules that define how the game is to be played.

Fair Share

FAIR SHARE

Suppose that s denotes a share of the booty S and that P is one of the players in a fair-division game with N players. We will say that s is a fair share to player P if s is worth at least $1/N$ th of the total value of S in the opinion of P . (Such a share is often called a proportional fair share, but for simplicity we will refer to it just as a fair share.)

Fair Division Methods

Thus, in a fair-division game we must consider not only the booty S and the players $P_1, P_2, P_3, \dots, P_N$ (each with his or her own opinions about how S should be divided), but also a specific method by which we plan to accomplish the fair division.

Fair Division Methods

There are many different fair-division methods known, but in this chapter we will only discuss a few of the classic ones. Depending on the nature of the set S , a fair-division game can be classified as one of three types: continuous, discrete, or mixed, and the fair-division methods used depend on which of these types we are facing.

Types of Fair Division Games

I. Continuous

In a **continuous** fair-division game the set S is divisible in infinitely many ways, and shares can be increased or decreased by arbitrarily small amounts.

Ex: the division of land, a cake, a pizza, and so forth.

Types of Fair Division Games

II. Discrete

A fair-division game is **discrete** when the set S is made up of objects that are indivisible

Ex: paintings, houses, cars, boats, jewelry, and so on.

(One might argue that with a sharp enough knife a piece of candy could be chopped up into smaller and smaller pieces. As a semantic convenience let's agree that candy is indivisible, and therefore dividing candy is a discrete fair-division game.)

Types of Fair Division Games

III. Mixed

A mixed fair-division game is one in which some of the components are continuous and some are discrete.

Ex: Dividing an estate consisting of jewelry, a house, and a parcel of land is a mixed fair-division game.

Types of Fair Division Games

Conclusion

Fair-division methods are classified according to the nature of the problem involved. Thus, there are discrete fair-division methods (used when the set S is made up of indivisible, discrete objects), and there are continuous fair-division methods (used when the set S is an infinitely divisible, continuous set). Mixed fair-division games can usually be solved by dividing the continuous and discrete parts separately, so we will not discuss them in this chapter.